$\vec{r}_A(t)$ designates the center-of-mass coordinate vector of Galaxy A and similarly, $\vec{r}_B(t)$ designates the center-of-mass coordinate vector of Galaxy B. Let us introduce now the center-of-mass vector $\vec{r}_{CM}(t) = \vec{r}_A(t) + \vec{r}_B(t)$ of the two galaxies, and their relative coordinate vector $\vec{r}(t) = \vec{r}_A(t) - \vec{r}_B(t)$ (Galaxy A and Galaxy B are assumed to have the same mass). We will work in the center-of mass coordinate system where $\vec{r}_{CM}(t) = 0$. Initially, at large enough separation, the center-of masses of the two galaxies move as pointlike particles. We want to put them on parabolic orbits in the x-y plane of the center-of-mass coordinate system. The following initial conditions are defined in dimensionless units:

(i) If the two galaxies were following the parabolic orbits throughout the collision, they would be found at t=0 with separation $r(t = 0) = R_0$ at the closest approach (pericenter) of the motion. At t=0, the point on the parabole representing the motion of the relative coordinate vector $\vec{r}$ is found at distance $r(0) = p/2 = R_0$ from the origin. Part of the initial condition is the constraint that the on the parabolic orbit of Galaxy A in the CM system $x_A(0) = z_A(0) = 0,\ y_A(0) = p/4$.

(ii) At $t = t_{init}$ the separation between A and B is R_{init}.

(iii) The default values are $M_A = M_B = 6.21200688$ for the equal masses of the two galaxies, if the Kuijken-Dubinski galaxy construction is used with default settings.

Introduce now the parameter η which is related to time by the nonlinear relation

$$t = \frac{1}{2}\sqrt{\frac{p^3}{2M}}\eta(1 + \frac{1}{3}\eta^2).$$

Show that the following relations hold for the parametrization of the relative cooordinates of the parabole of the galaxy pair:

(a) $|\vec{r}(\eta)| = r \ = \ \frac{1}{2}p(1 + \eta^2),$

(b) $x(\eta) \ = \ p\eta,$

(c) $y(\eta) \ = \ \frac{1}{2}p(1 - \eta^2).$

Calculate the initial coordinates and velocities of Galaxy A and Galaxy B in the center-of-mass coordinate system. Provide the numerical values for $R_0 = 2.5,\ R_{init} = 44.$