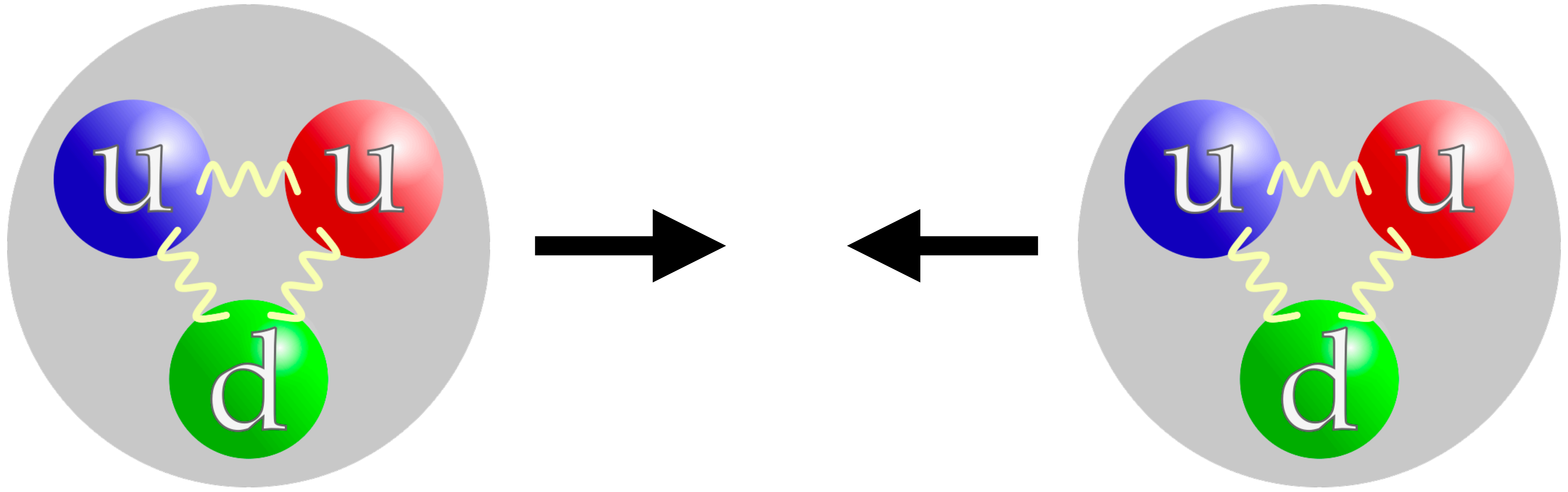
PHYS 142/242

Lecture 19: Particle Physics & VEGAS (Continued)

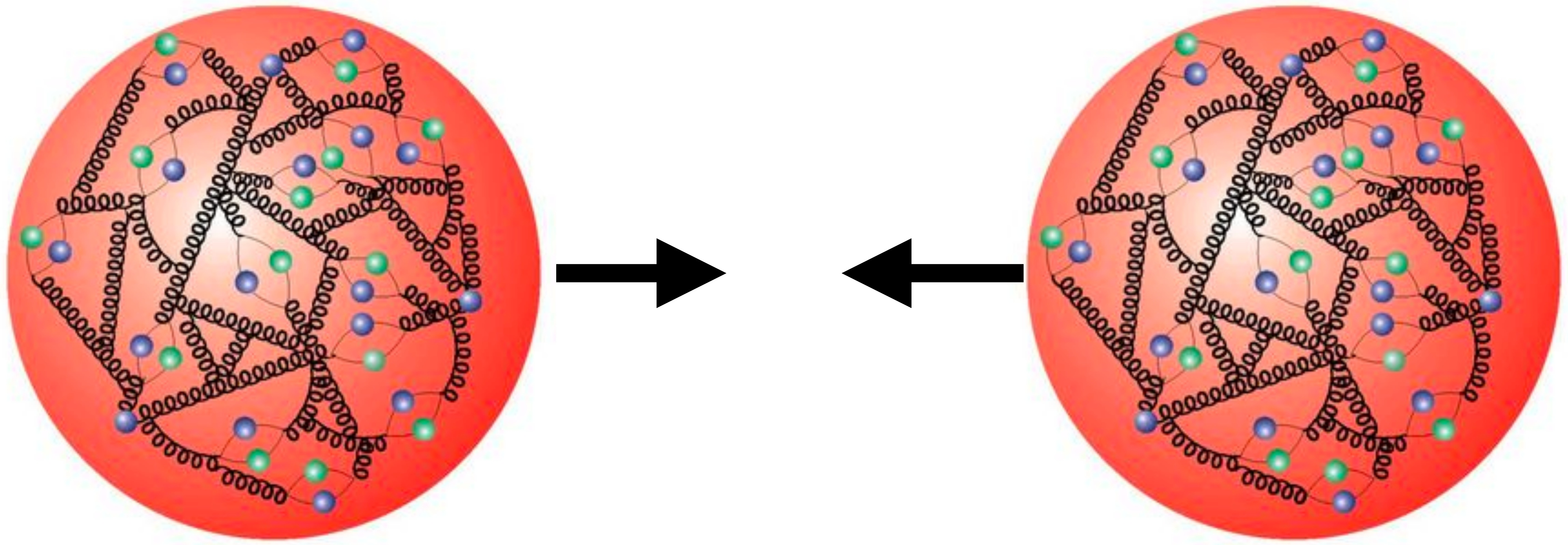
Javier Duarte — February 26, 2025

Motivation

In high energy particle physics, we often want to calculate what happens when we collide two protons



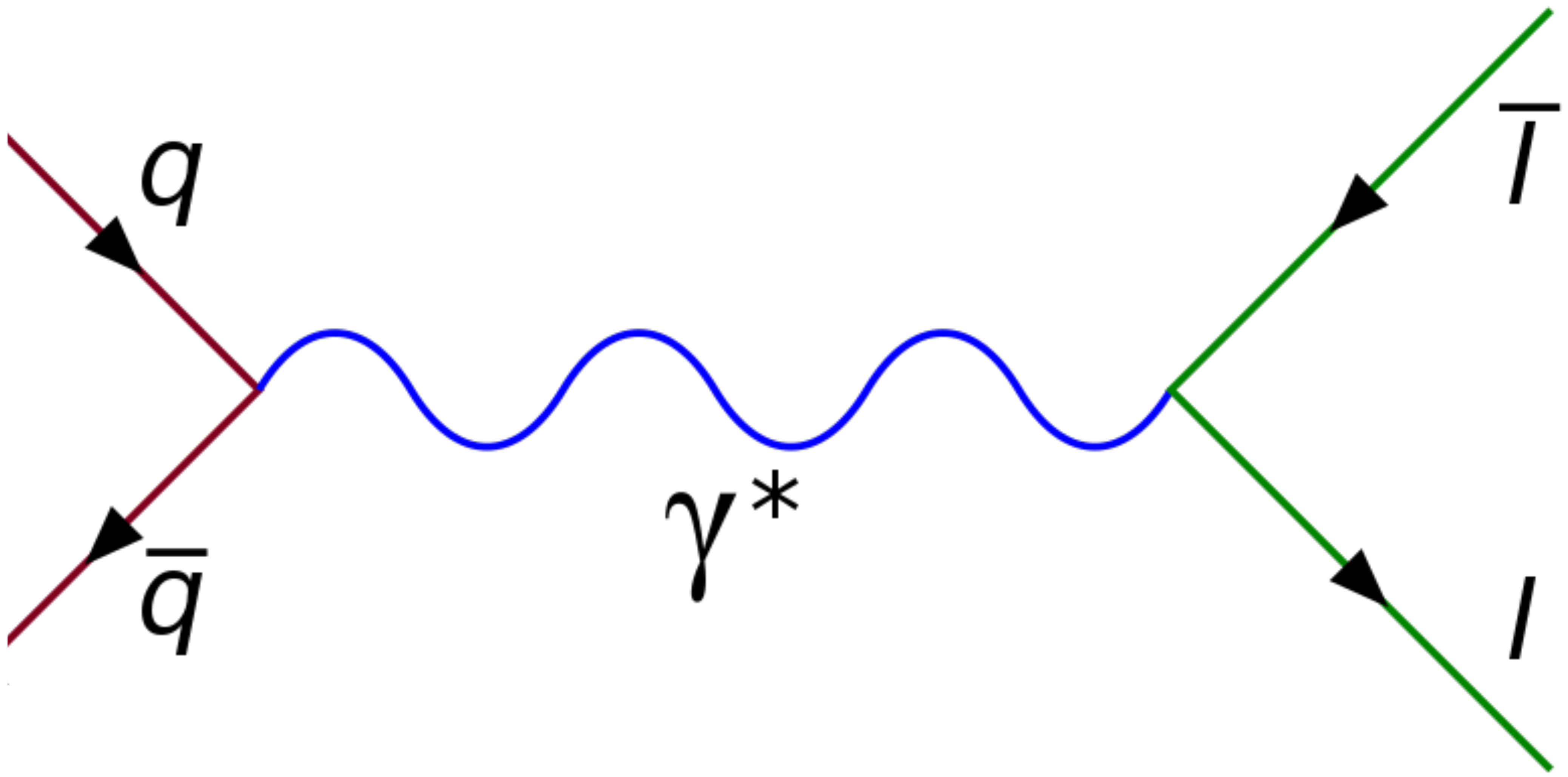
But protons are actually filled with a lot of "stuff" (quarks and gluons, collectively called partons)



Drell-Yan production

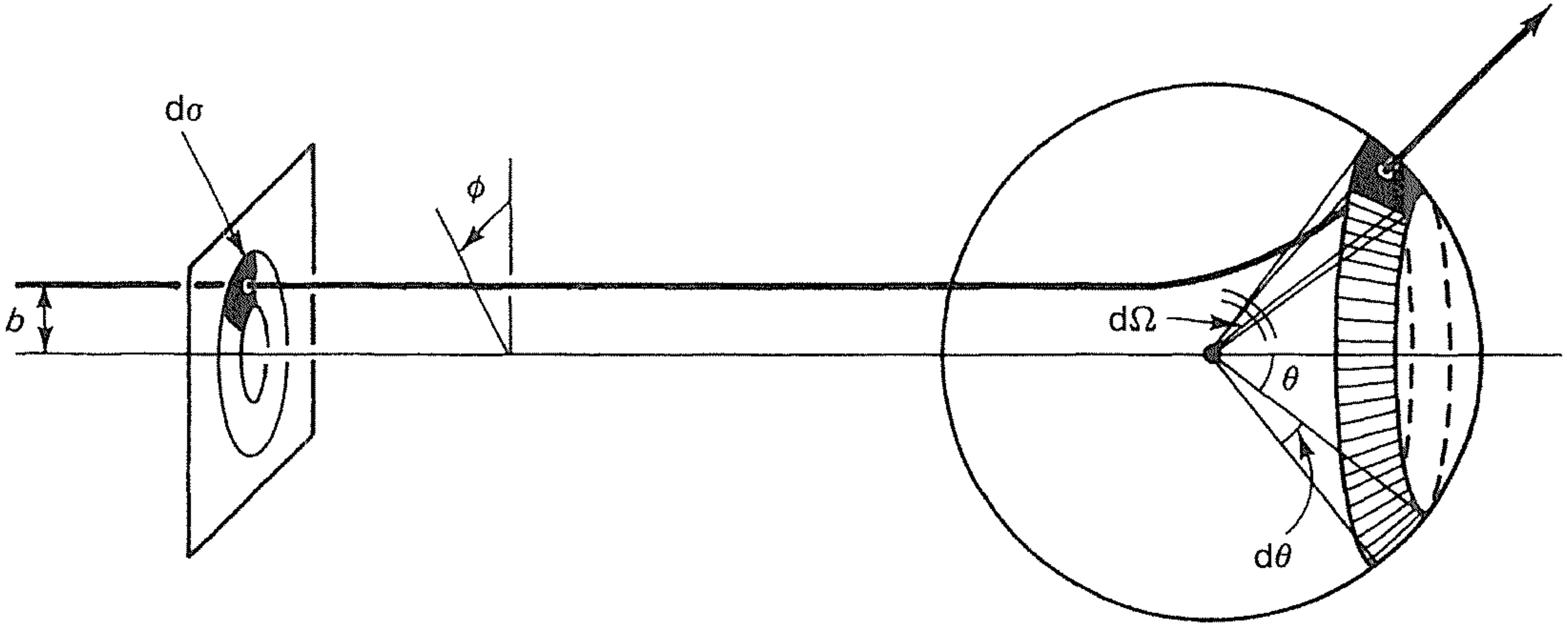
We can calculate using a "path integral" approach: sum/integrate over all the possible ways go from the initial state (two quarks) to the final state (two leptons)

Feynman rules tell us the amplitude $\mathscr{M}$ for each "path" which can be represented by a diagram



Cross section

Differential scattering cross section $\frac{d\sigma}{d\Omega}$ usually depends on the scattering angle θ



Total cross section is given by integral over solid angle

$$\sigma = \int d\Omega \frac{d\sigma}{d\Omega} = \int d\theta d\phi \sin\theta \frac{d\sigma}{d\Omega}$$

Cross section can be thought of as the probability for a given process to occur

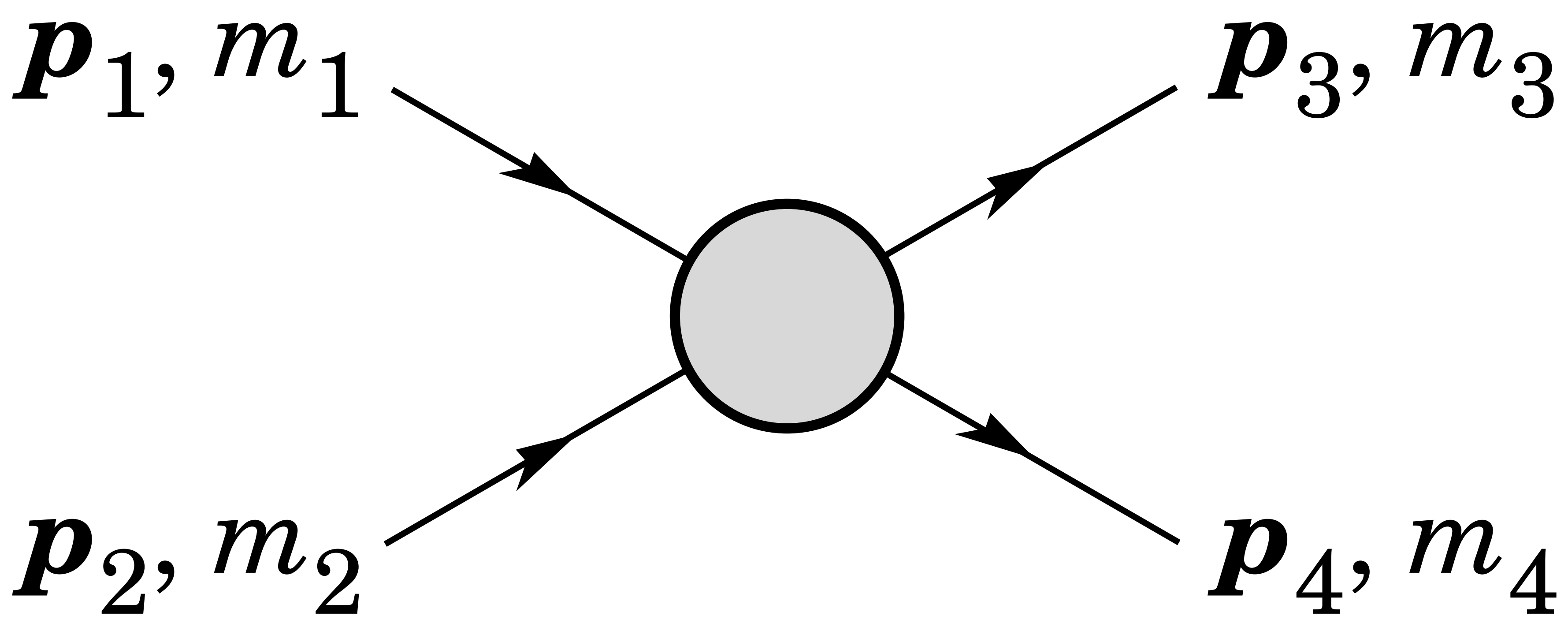
Fermi's Golden Rule (for scattering)

Ingredients: amplitude ($\mathscr{M}$) for the process and the phase space (Ω) available

For a two-to-two scattering process ($1+2\to 3+4$), cross section is given by

$$\sigma = \frac{S}{4\sqrt{(p_1\cdot p_2)^2-(m_1m_2)^2}}\int |\mathscr{M}|^2(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)$$

$$\times\prod_{j=3}^{4}2\pi\delta(p_j^2-m_j^2)\theta(p_j^0)\frac{d^4p_j}{(2\pi)^4}$$



S accounts for double-counting with identical particles

Each outgoing particle lies on its mass shell

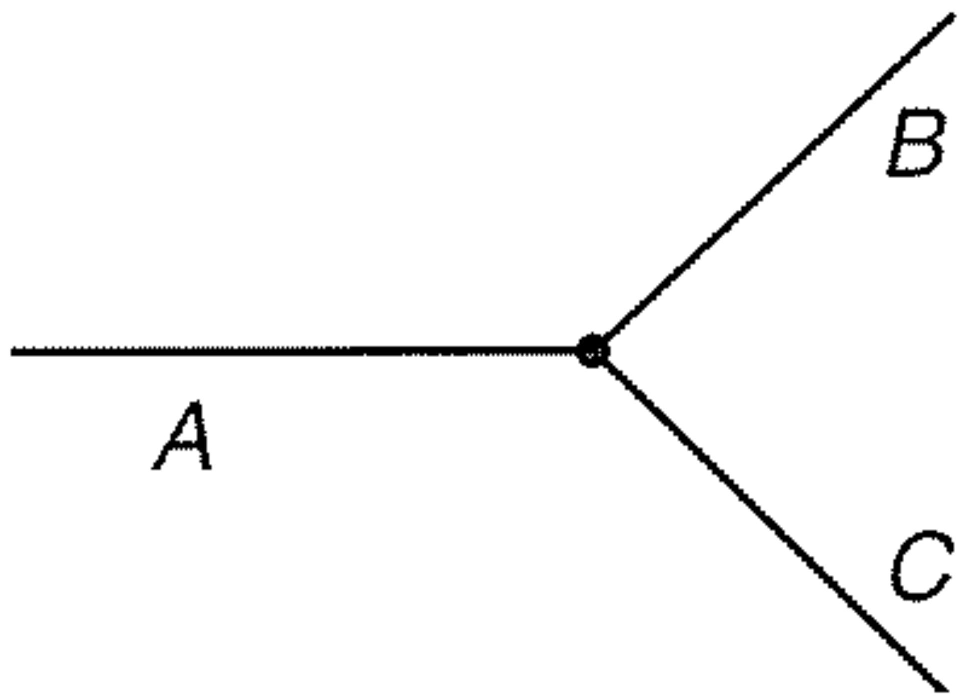
Each outgoing energy is positive

Energy and momentum must be conserved

Although we can't fully calculate σ without knowing the form of $\mathscr{M}$, we can calculate the differential cross section:

$$\frac{d\sigma}{d\Omega}=\left(\frac{1}{8\pi}\right)^2\frac{S|\mathscr{M}|^2}{(E_1+E_2)^2}\frac{|\boldsymbol{p}_f|}{|\boldsymbol{p}_i|}$$

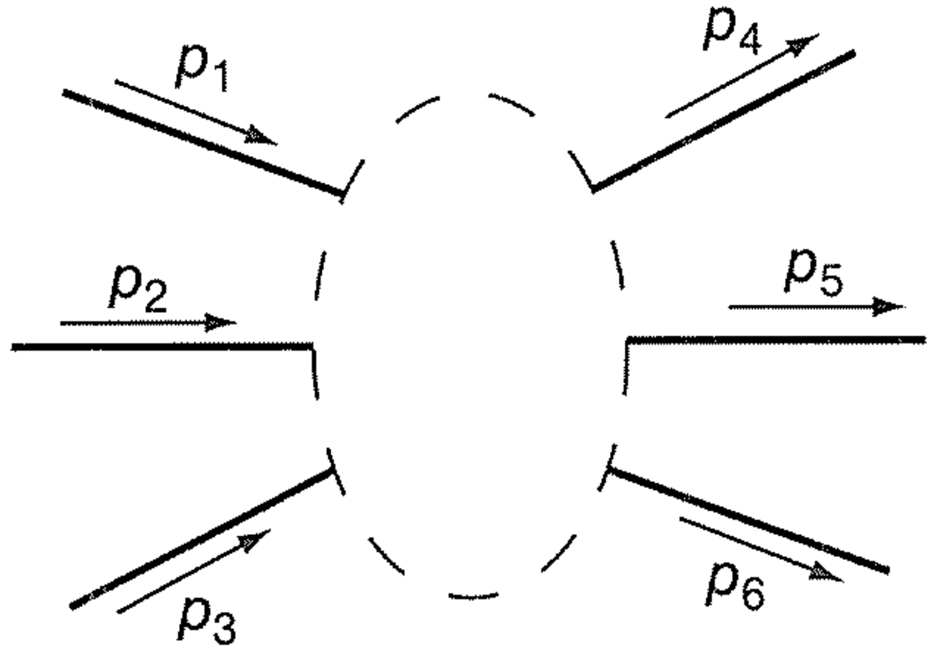
Feynman rules for ABC toy theory



1. Label incoming/outgoing 4-momentum $p_1, p_2, \ldots, p_n$ and internal 4-momenta $q_1, q_2, \ldots$

2. Vertex factors: For each vertex, write $-ig$

3. Propagators: For each internal line, write $\dfrac{i}{q_j^2 - m_j^2}$

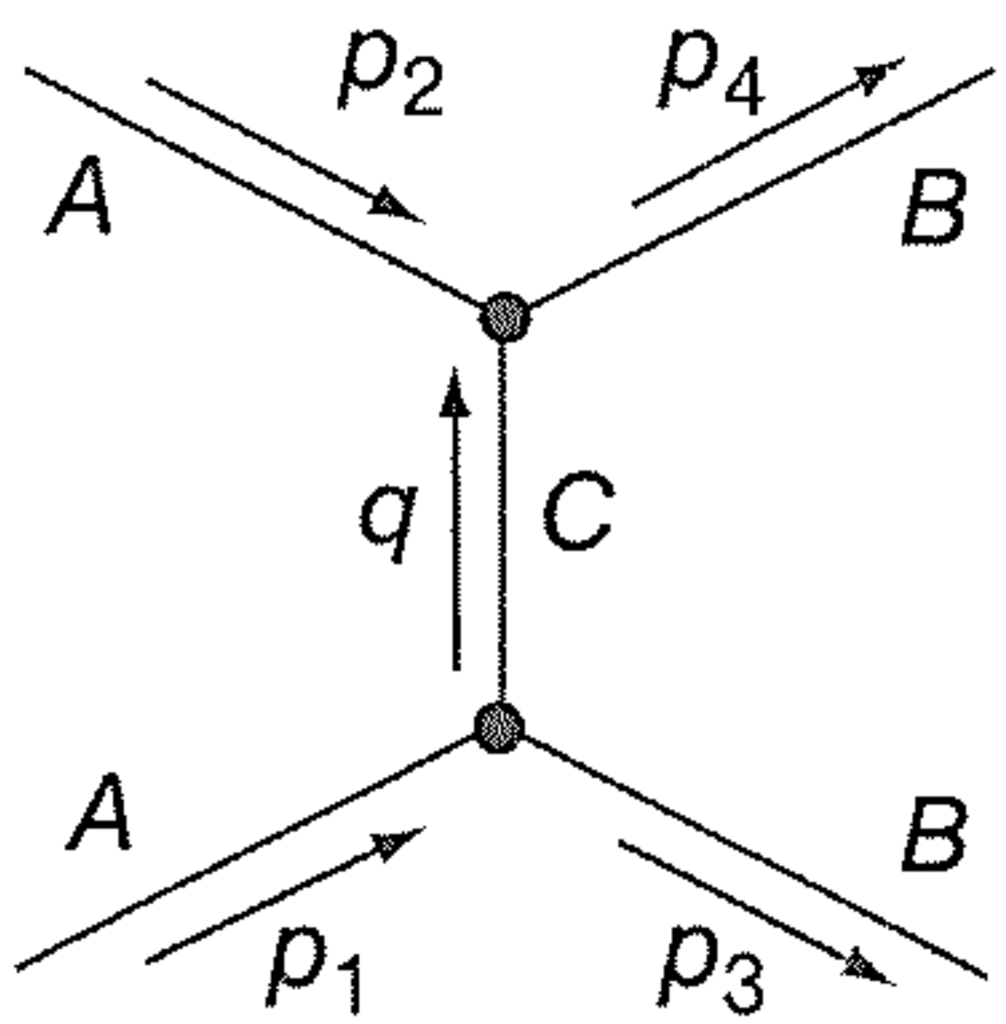


4. Conservation of energy & momentum: For each vertex, write $(2\pi)^4\delta^4(k_1 + k_2 + k_3)$

5. Integrate over internal momentum: For each internal line, write $\dfrac{1}{(2\pi)^4} d^4 q_j$

6. Cancel the delta function $(2\pi)^4\delta^4(p_1 + p_2 + \ldots + p_n)$ and multiply by i to get $\mathscr{M}$

Scattering: $A + A \to B + B$

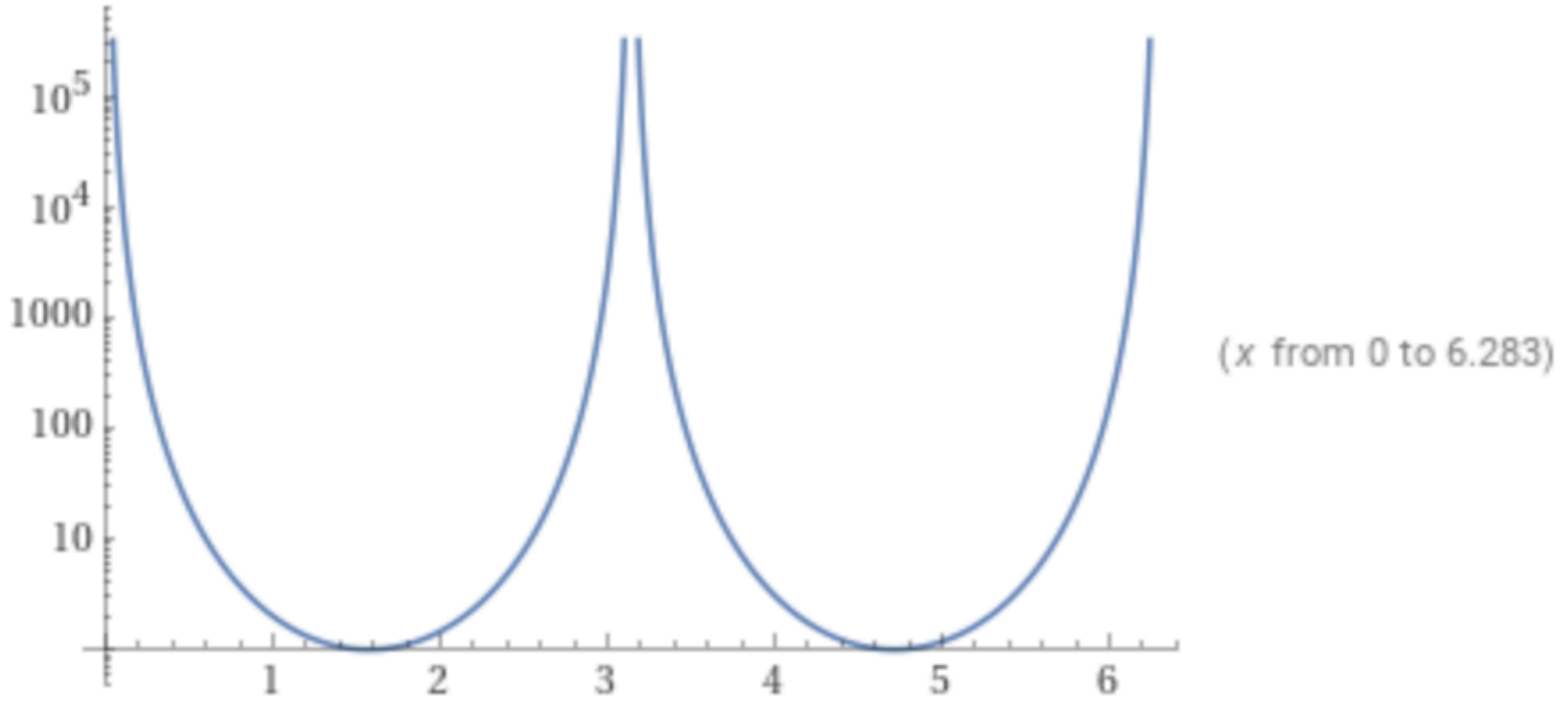


Matrix element is

$$\mathscr{M} = -\frac{g^2}{\boldsymbol{p}^2 \sin^2\theta}$$

Differential cross section is

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\left(\frac{g^2}{16\pi E\boldsymbol{p}^2 \sin^2\theta}\right)^2$$



To get the total cross section, we would integrate this

$$\sigma = \int d\Omega \frac{d\sigma}{d\Omega} = \int d\theta d\phi \sin\theta \frac{d\sigma}{d\Omega}$$